Towards Automatic Numerical Cross-Checking: Extracting Formulas from Text

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ABSTRACT

Verbal descriptions over the numerical relationships among some objective measures widely exist in the published documents on Web, especially in the financial fields. However, due to large volumes of documents and limited time for manual cross-check, these claims might be inconsistent with the original structured data of the related indicators even after official publishing. Such errors can seriously affect investors' assessment of the company and may cause them to undervalue the firm even if the mistakes are made unintentionally instead of deliberately. It creates an opportunity for automated Numerical Cross-Checking (NCC) systems. This paper introduces the key component of such a system, formula extractor, which extracts formulas from verbal descriptions of numerical claims. Specifically, we formulate this task as a DAG-structure prediction problem, and propose an iterative relation extraction model to address it. In our model, we apply a bi-directional LSTM followed by a DAG-structured LSTM to extract formulas layer by layer iteratively. Then, the model is built using a human-labeled dataset of tens of thousands of sentences. The evaluation shows that this model is effective in formula extraction. At the relation level, the model achieves a 97.78% precision and 98.33% recall. At the sentence level, the predictions over 92.02% of sentences are perfect. Overall, the project for NCC has received wide recognition in the Chinese financial community.

ACM Reference Format:
Yixuan Cao[1,2], Hongwei Li[1,2], Ping Luo[1], Jiaquan Yao[3] [1]Key Lab of Intelligent Information Processing of Chinese Academy of Sciences (CAS), Institute of Computing Technology, CAS, Beijing 100190, China [2]University of Chinese Academy of Sciences, Beijing 100049, China [3]School of Management, Jinan University, Guangzhou 510632, China {caoyixuan, lihongwei, luop}@ict.ac.cn, {jiaquanyao}@gmail.com, . 2018. Towards Automatic Numerical Cross-Checking: Extracting Formulas from Text. In *WWW 2018: The 2018 Web Conference, April 23–27, 2018, Lyon, France.* ACM, New York, NY, USA, 10 pages. https://doi.org/10.1145/3178876.3186166

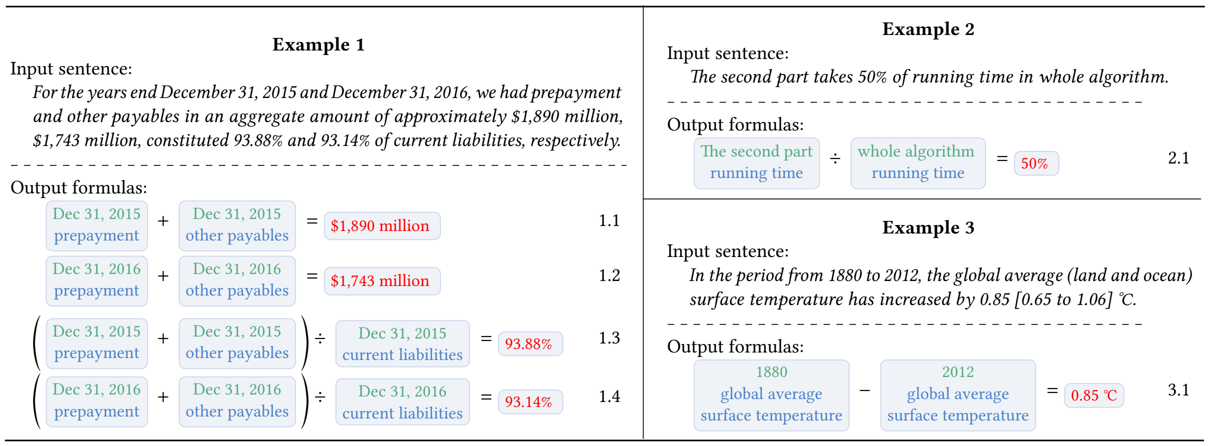
1 INTRODUCTION

Claims over the numerical relationships among some objective measures widely exist in the published documents on Web. For example, various financial documents (e.g. IPO prospectus, bond prospectus, corporate annual report etc.) contain large fraction of verbal descriptions over the finance indicators of the corporates. Example 1 in Fig. 1 is such a typical sentence. It describes the amount of sum of *prepayment* and *other payables* for a company at two time point (the end of 2015 and 2016), and also the share of this sum in *current liability.* Also, Fig. 1 includes another two examples of such verbal descriptions from the fields of natural sciences (computer science and meteorology, respectively). All these sentences aim to verbally describe the "accurate" numerical relationships of some objective indicators in a quantitative way.

Even though these claims of numerical relationships are published officially, they might be inconsistent with the structured data of the indicators, which generate the verbal description. See a real-world example for this inconsistency in Fig. 2, which is detected automatically by our proposed system. From the given sentence, we can get the formula related to the indicators. Note that it is only one of the formulas which exactly match the semantics of the sentence. The document, which includes this sentence, also contains a table of these finance indicators. Putting these original data from table into the left side of the formula, we find that *prepayment* and *other payables* aggregately constituted 91.13% of *current liability* in 2015. This conflicts with the number 93.88% given in the text.

Since the essential requirements for disclosure documents, especially in the financial areas, are "authenticity, accuracy and completeness", these numerical errors might bring about huge reputation risk, and even economic losses. In 2011, Goldman Sachs made one critical typo in several-hundred pages of filings – wrote "×" when they wanted "/" . That caused the trading price spiking and the trading was eventually suspended. This typo resulted in $45 million loss for Goldman[1]. Recently, in 2016 Postal Savings Bank of China (PSBC) announced its annual report, in which the *total liability* was written to "￥80,000 billion" whereas the actual value should be "￥8 billion". The news spread online rapidly and severely harmed the reputation of PSBC among investors[2]. Since these disclosure documents usually have the force of law, these errors and typos should be thoroughly removed before publishing.

[1]http://www.economist.com/node/18744559?story_id=18744559
[2]http://www.cfi.net.cn/p20161223000079.html



Example 1

Input sentence:

For the years end December 31, 2015 and December 31, 2016, we had prepayment and other payables in an aggregate amount of approximately $1,890 million, $1,743 million, constituted 93.88% and 93.14% of current liabilities, respectively.

Output formulas:

$\boxed{\text{Dec 31, 2015 prepayment}} + \boxed{\text{Dec 31, 2015 other payables}} = \boxed{\text{\$1,890 million}}$ 1.1

$\boxed{\text{Dec 31, 2016 prepayment}} + \boxed{\text{Dec 31, 2016 other payables}} = \boxed{\text{\$1,743 million}}$ 1.2

$\left(\boxed{\text{Dec 31, 2015 prepayment}} + \boxed{\text{Dec 31, 2015 other payables}}\right) \div \boxed{\text{Dec 31, 2015 current liabilities}} = \boxed{93.88\%}$ 1.3

$\left(\boxed{\text{Dec 31, 2016 prepayment}} + \boxed{\text{Dec 31, 2016 other payables}}\right) \div \boxed{\text{Dec 31, 2016 current liabilities}} = \boxed{93.14\%}$ 1.4

Example 2

Input sentence:

The second part takes 50% of running time in whole algorithm.

Output formulas:

$\boxed{\text{The second part running time}} \div \boxed{\text{whole algorithm running time}} = \boxed{50\%}$ 2.1

Example 3

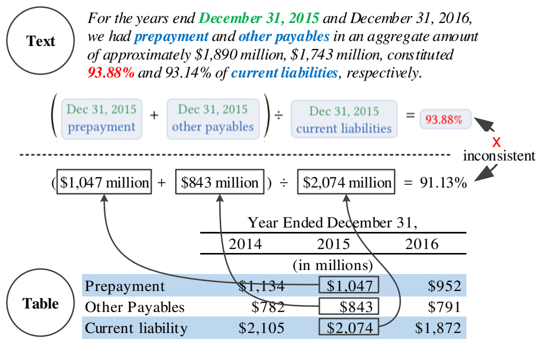
Input sentence:

In the period from 1880 to 2012, the global average (land and ocean) surface temperature has increased by 0.85 [0.65 to 1.06] ℃.

Output formulas:

$\boxed{\text{1880 global average surface temperature}} - \boxed{\text{2012 global average surface temperature}} = \boxed{0.85\ ℃}$ 3.1

Figure 1: Examples of sentences and their formulas.



Text
*For the years end **December 31, 2015** and December 31, 2016, we had **prepayment** and **other payables** in an aggregate amount of approximately $1,890 million, $1,743 million, constituted **93.88%** and 93.14% of **current liabilities**, respectively.*
(Dec 31, 2015 prepayment + Dec 31, 2015 other payables) ÷ Dec 31, 2015 current liabilities = 93.88%
x
inconsistent
($1,047 million + $843 million) ÷ $2,074 million = 91.13%
Table

	Year Ended December 31,		
	2014	2015	2016
		(in millions)	
Prepayment	$1,134	$1,047	$952
Other Payables	$782	$843	$791
Current liability	$2,105	$2,074	$1,872

Figure 2: Numerical Cross-Checking detects inconsistency between verbal description and original data.

It well documented in finance literature that information in the disclosure documents is always one of the greatest concerns to the investors. Lawrence [14] shows that individual investors invest more in firms with clear and concise financial disclosure, since such disclosures reduce individuals' relative information disadvantage. Choudhary et al. [7] point out that even immaterial errors in the documents contain information about firm's financial reporting reliability. Hence, accounting errors may cause the investors to lose trust in the firms reporting quality, and decrease investments in these firms. Besides, investors' attention to the accounting errors can lead to damage to firms' reputation.

To this aim, there is a special job called "authorized reading" to conduct *Numerical Cross-Checking* manually. Since the original data tables are updated frequently during writing, the first draft of disclosure often contains many data-inconsistency errors. Based on user interview of 10 Chinese investment bankers, on average it takes one employee with 3-year work experiences one week for the task of cross-check over a 500-page document. Additionally, there is usually a hard deadline to publish the disclose documents, thus the time left for cross-check is limited. More importantly, conducting cross-check for a long period of time definitely induces fatigue, tiredness, and carelessness. Thus, even after manual cross-check, these data-inconsistency errors are still inevitable due to the large volumes of documents, frequent updates of the original table, limited time for cross-check, and largely the fatigue resulted from this intellectually demanding, laborious, time-consuming process.

Therefore, this challenge creates an opportunity for automated Numerical Cross-Checking (NCC) systems. As there is simply no existing system that truly does this automatically, NCC technology is clearly falling behind. There are some related systems developed. ClaimBuster [10] is a fact-checking system that aims to automatically check important factual claims, especially claims in political discourses. Technically, it focused on detecting check-worthy factual claims while the other two components of matching claims and checking claims are still ongoing. Currently, it cannot support precise cross-check over numerical indicators. StatCheck [20] uses rule-based program to check inconsistency errors in the null-hypothesis significance testing, presented in the academic papers in major psychology journals. It finds that one in eight papers contains a grossly inconsistent p-value that may have affected the statistical conclusion. While the relevant tools and techniques can assist this task in various steps, a full-fledged, end-to-end solution to NCC does not exist.

To fill this gap, w e are building an end-to-end system for computer-assisted Numerical Cross-Checking which uses machine learning, natural language processing, and database query techniques to automate this task. It expects a financial document as input and detects all potential conflicts in numerical relationships, where Fig. 2 is a typical example output by this system. In an evaluation over 1,000 officially published Chinese prospectus, our system found that 68.92% of them contain data-inconsistency errors, and on average each document with inconsistency has 4.26 errors (confirmed by professionals' re-check).

The system has three major components: formula extraction (extract numerical relationships from text), table extraction (extract data from tables), and consistency checking. While the improvement of the full-fledged system is still ongoing, in this paper we focus on the key component of the system, formula extractor, which extracts formulas from verbal descriptions of numerical claims.

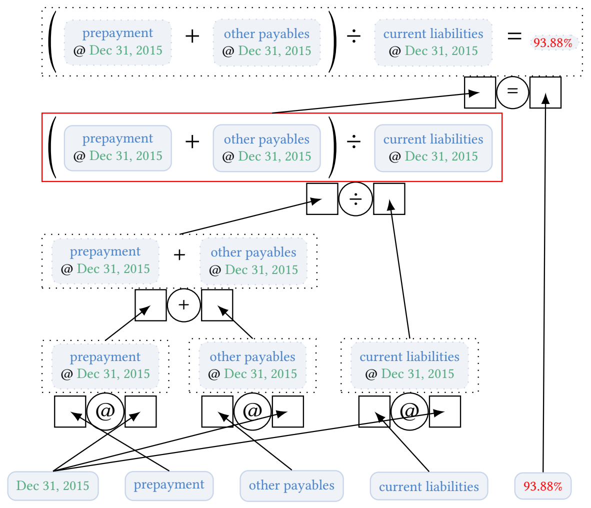

prepayment @ Dec 31, 2015
other payables @ Dec 31, 2015
current liabilities @ Dec 31, 2015
93.88%
Dec 31, 2015
prepayment
other payables
current liabilities
93.88%

Figure 3: The illustration of iterative relation extraction by example (Formula 1.3 in Fig. 1).

Given the plethora of discourses and narratives exposed on Web, this component extracts the formulas, which are semantically expressed by the claim sentences. Fig. 1 shows some typical examples of input and output of this component. This is essentially the key step to automate the cross-check process.

We propose an iterative relation extraction (IRE) model to extract formulas from text. Fig. 3 illustrates how Formula 1.3 in Fig. 1 is converted into a directed acyclic graph (DAG) structure. In this graph each node refers to a binary relation, whose type can be any *computation relation* (say $+, -, \times, \div$ etc.) or any *compare relation* (say $=, <, >$ etc.). The two operands for a relation can be entities (see the leaves in the bottom), or the relations at the lower layers. See the node in the red solid rectangle as an example. It is a relation of $\div$ with two operands at the lower layers. Its left operand refers to the node with a $+$ relation, representing the sum of *prepayment* and *other payables* (at the end of 2015), while its right operand refers to a "@" node [3], representing the value of *current liability* at 2015 year end. In this case the two operands are both the relations generated at the lower layers. Also, see the root node with a "=" relation, with the left operand of "$\div$" and the right one of the leaf entity 93.88%. This root node actually represents the whole formula. Thus, any formula can be represented as DAG, in which each internal node has two children and the nodes at the lower layers can be the children of the nodes at the upper layers. It means that the relations generated at the lower layers can be recursively used as the operands of some new relations at the upper layers. This is why we call this process iterative relation extraction.

Previous studies of relationship extraction mostly focus on the relations among entities, thus they only perform relationship extraction for a single layer. In the proposed IRE model, it extracts relations from bottom to top iteratively until a formula is generated finally. In this process the resultant relations extracted at the lower layers can be used as the input of the relationships at the upper layers. In this way, the layer-by-layer iterative computing process generates the ultimate formula.

[3] The meaning of @ will be detailed in Section 2

In this paper we develop the neural-based method for IRE. In this model we consider the sequential information in two directions, namely horizontally and vertically. In horizontal direction we consider the word sequence of the original input sentence. In vertical direction we model the sequences from leaves to root in the DAG for formula generation. Specially, we use two different LSTM modules to model the sequential information in these two directions so that the building of a relation depends on not only the relations generated earlier, but also the semantics embedded in the original sentence. Finally, the model is trained on a labelled dataset of tens of thousands of sentences, which are collected from Chinese public financial documents. The evaluation result shows that our model archives at 97.78% precision and 98.33% recall at relation level. And on 92.02% of sentences, it predicts the formulas without any mistakes. Additionally, to accelerate the training process, two techniques are developed: *parallel by layer* and *parallel by batch*. With these two techniques the average training time for one epoch speeds up more than 5 times. These techniques might shed some light on speeding up models with tree or DAG structure inputs.

Last but not least, our study makes contribution to the measurement error problem in the proxies of accounting errors and reporting bias by introducing more precise proxy of accounting errors. This problem is currently a big concern and remains unsolved in the literature. Researchers tend to rely on restatement data to construct proxies for accounting errors, and focus on examining the possible reasons and consequences of these errors. However, there exists measurement error problem in the existing proxies, which is caused by the limited sample size of restatement data [8]. The errors undetected or detected but untracked by the restatement data can lead to underestimating the incidence of accounting errors. With NCC system, we are able to find out errors in a much larger scope of disclosure documents, which makes us to estimate accounting errors more precisely.

The rest of the paper is structured as follows: Section 2 gives a definition of formula and its structure. Section 3 describes our iterative relation extraction model. Then we evaluate our model in Section 5, summarize related work in Section 6 and conclude in Section 7.

2 STRUCTURE OF FORMULAS IN TEXT

In this section we will first define the structure of one formula, then extend that to the structure of all formulas in one sentence, and finally discuss the characteristics of the structure. The discussion in this section is based on Fig. 4. Notice that Fig. 3 is a subgraph of Fig. 4 (nodes with solid border). And we use the operator to represent each relation instead of long expression for brevity.

2.1 Structure of One Formula

Intuitively a *formula* is a formatted expression about something equal, or greater / less than another thing. For example, revenue at 2017 equals some amount of money; the growth rate of revenue from 2016 to 2017 equals a number. A formula is constructed by components including entities and relations. We will introduce these components first and then define the structure of a formula.

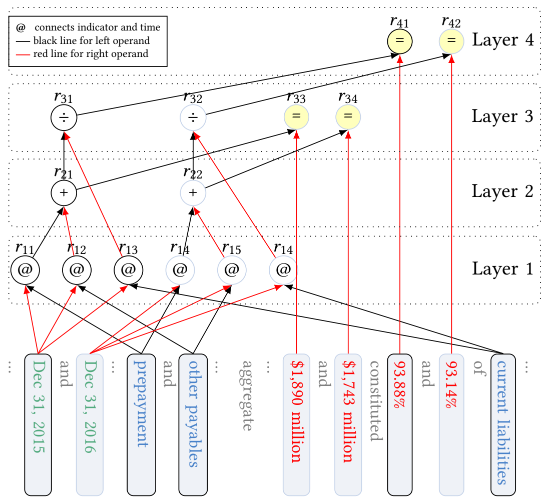


Figure 4: Structure of formulas from Example 1 in Fig. 1.

Entity. In the bottom row of Fig. 4, the gray boxes are the basic components of formula: *entities*. An entity is a single word or a phrase that participate in a formula. We define three types of entities in financial text: *time* entities like "January 31, 2015", finance *indicator* like "revenue", "current liabilities", and *values* like "$166.0 million", "93.88%". Time and value entities are extracted by handcrafted regular expressions. Finance indicators are predefined in a large whitelist maintained by financial specialists. We extract them by matching the whitelist. From now on we assume these three types of entities are given.

Relation. From Layer 1 to layer 4 in Fig. 4, we see patterns that two nodes in lower layer point to one node in upper layer. That structure is called *relation*.

Definition 2.1. **Relation.** A relation consists of three parts: left operand c_l, right operand c_r, and *operator* o:

$$r : (c_l, o, c_r) \tag{1}$$

where each operand c_* could be an entity or another relation, and o is an operator like "+" chosen from a predefined operator set.

Notice on two things. First, the definition of relation is *recursive*: the two operands of a relation can be other relations. Second, the left operand and the right operand are not commutative ($a - b$ is clearly different with $b - a$). Although some operators such as "+" and "=" are commutative mathematically, we make rules to force all operators to be non-commutative. For example, in Fig 4, r_{41} has two operands r_{31} and 93.88%. The first word in r_{31} "Dec 31, 2015" appears earlier than 93.88% in sentence. We set the rule that $(r_{31}, =, 93.88\%)$ is a valid relation, however $(93.88\%, =, r_{31})$ is not. Operator @ connects an indicator with the time it refers to. The left operand of @ must be an indicator and the right operand must be a time. Otherwise the relation is not valid.

In this study the used relations are divided into two categories: arithmetic and comparison.

Arithmetic relation. An arithmetic relation is a relation with an arithmetic operator from set $O_a : \{@, +, -, \times, \div\}$. An arithmetic relation describes a component of a formula. For example, $r_{11} : (prepayment, @, Dec31, 2015)$ describes an indicator at 2015, $r_{21} : (r_{11}, +, r_{12})$ describes a summation of two indicators in 2015. All the arithmetic relations will be used as operands of the relations in higher levels. Thus, we call them *intermediate* nodes.

Comparison relation. A comparison relation is a relation with a comparison operator from set $O_c : \{=, >, <\}$. A comparison makes a statement and cannot be an operand of another relation. For example, r_{41} states r_{31} equals 93.88%. Since the comparison relations will not be used as the operands of other relations, we call them *terminal* nodes.

Using the definitions above, a *formula* is a structure consisting of relations and entities. With one comparison at the top, all the descendants, including arithmetic relations and entities, construct a formula.

2.2 Formulas in a Sentence

But one sentence might express multiple formulas whose structures are entangled together. This makes the structure of all formulas in a sentence more complex in several ways.

First, The structure of formulas in one sentence might has multiple roots, such as Fig. 4 illustrated. Second, one entity or relation could point to multiple relations, that is, the same occurrence of entities and relations can be *reused* multiple times by other relations. For example, "Dec 31, 2015" is the right operand of r_{11}, r_{12}, and r_{13}; r_{21} is the left operand of both r_{31} and r_{33}.

Layer. The structure is also layered. The dotted lines groups relations by layer. A layer of relations is all relations that have the same layer number. The layer number of a relation $r : (c_l, o, c_r)$ is defined as $l(r) = \max(l(c_l), l(c_r)) + 1$. We set $l(c_*) = 0$ if c_* is an entity.

From the definition and analysis above, the structure of formulas in one sentence has the following characteristics:

(1) a formula is defined in a recursive way from the bottom components: time, indicator and values, through several layers of nested relations, to a comparison;
(2) it may have multiple roots;
(3) relations and entities could be reused; and
(4) every relation has two ordered operands and one operator.

The first three characteristics match exactly with the DAG (Directed Acyclic Graph) structure. The fourth characteristic constrains the DAG to an ordered, binary DAG, where each node is associated with an operator. Here the ordered binary DAG means that every node has exactly two non-commutative children.

Therefore, the problem of extracting formulas in a sentence is defined as extracting the ordered binary DAG structure of formulas.

3 FORMULA EXTRACTION

Motivated by the recursive property of formula, we come up with an iterative relation extraction (IRE) model to extract formulas layer by layer iteratively, from bottom to top. In this section, we first give an overview of our model. Then, we dive into all the modules, including the embedding, Bi-LSTM, and DAG-LSTM modules, to introduce the details.

3.1 Framework of IRE Model

The key idea of the IRE model is extracting relations layer by layer iteratively. It extracts one layer of relations by first generating relation candidates and then doing binary classifications. When it terminates, the nodes with positive labels from all layers constitute the formulas. The process is as follows.

Store the extracted time, indicator and value entities in sets T, I, V respectively. Let operator set $O = \{@, +, -, \times, \div, >, <, =\}$. Use set N to record the nodes in the DAG structure. At the beginning, $N = T \cup I \cup V$.

First layer. First, generate all the possible candidates:

$$C = \{(n_1, o, n_2) | n_1, n_2 \in N, o \in O\} - N. \tag{2}$$

Then, predict all candidates in C. Suppose the nodes with positive labels are N_1^+, update $N = N \cup N_1^+$.

Repeat. Afterwards, we repeat the process in the the first layer, where at layer i the candidate set is generated using Eq. 2, but with updated N. Note that for a candidate of layer i, the operands come from N, which includes relations and entities of all lower layers $k, k < i$. The process terminates when there is a layer j, $N_j = \emptyset$, and N contains the extracted structure of formulas.

In our work, we constrain that the first layer only contain the relations of "@", and all the other layers do not include "@" relations.

Algorithm 1 Model framework

```
Extract time entities $T$, indicator entities $I$, and value entities $V$.
Use subscript $a$ for arithmetic operator or relation, $c$ for comparison operator or relation.
Operators $O_a = \{@, +, -, \times, \div\}$, $O_c = \{>, <, =\}$
Entities and arithmetic relations $R_a = T \cup I \cup V$
Comparison relations $R_c = \emptyset$
repeat
    $C_a = \{(r_1, o, r_2) \mid r_1, r_2 \in N, r_1 \neq r_2, o \in O_a\} - N$
    $C_c = \{(r_1, o, r_2) \mid r_1, r_2 \in N, r_1 \neq r_2, o \in O_c\} - R_c$
    $R_a^+ = \text{predict}(C_a)$; $R_c^+ = \text{predict}(C_c)$
    $R_a = R_a \cup R_a^+$; $R_c = R_c \cup R_c^+$
until $R_a^+ = \emptyset$
return $R_a \cup R_c$
```

Algorithm 1 describes this process in detail. And we demonstrate this process by an example in Fig. 5. The input sentence is shown at the bottom. At each layer, the bottom row includes current entities and relations, and boxes above them are candidates. Each column (in the dotted rectangle) contains all operators between two operands (we only draw arrows on the operator "−" for brevity). Black arrow connects relation with its left operand, while red with the right operand. Positive candidates are highlighted in green. And the dashed arrows from the green boxes show that these nodes will be used as the operands in the higher layers. The process terminates after Layer 3 since no arithmetic candidates are positive, and thus no more candidates could be generated further.

In our work, the `predict` function is a neural network consists of three modules (Fig. 6): embedding module, Bi-LSTM module and DAG module. In the horizontal direction, the embedding module embeds words into high dimensional dense vectors, and the Bi-LSTM module informs each word its context. In the vertical

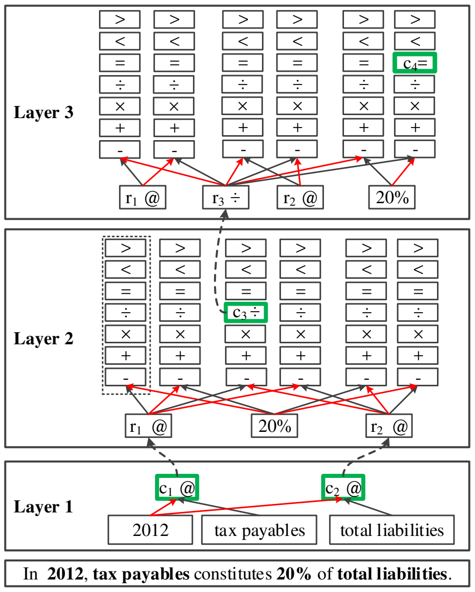


Figure 5: The framework of IRE, illustrated by an example.

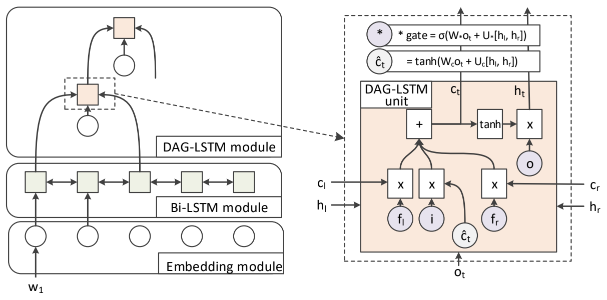


Figure 6: Our iterative relation extraction model. Overview on left and details of DAG-LSTM unit on right.

direction, the DAG-LSTM module extract relations iteratively from leaves to roots. All these three module are trained together and their parameters are updated simultaneously. The details are described below.

3.2 Embedding module and Bi-LSTM module

The embedding module contains a look-up table that maps each token in a sentence to a distributional representation: $(e_1, e_2, ..., e_n) = E(w_1, w_2, ..., w_n)$. Since each e_i is isolated from each other, we feed them into a Bi-LSTM network to connect them together. For each w_i, Bi-lSTM concatenates the hidden states of forward and backward uni-directional LSTMs as its hidden state h_i. We can assume that h_i contains long-term and short-term context information of w_i [9, 22], which make the next step, namely growing the DAG structure from each token's hidden state, possible.

3.3 DAG-LSTM module

In this paper, we consider only one type of DAG structure – ordered binary DAG. It can be to extend to other types of DAG [25].

We introduce this module by the feed-forward process, the process to predict formulas in a sentence. The key idea is to predict one layer a time iteratively which has been discussed in Algorithm 1. Now we focus on the DAG computation part.

Operator Emedding. The operator is a very important component of a relation. For example, a division relation between two revenues might be compared with a fraction or percentage later, but should not be compared with some amount of money. On the contrary, if you only know two revenues have a relation, the information is ambiguous. Since entities are representing by hidden states, we make a special embedding matrix $\boldsymbol{E_o}$ to embed each operator o into a distributional representation $o_t = \boldsymbol{E_o}(o)$. $\boldsymbol{E_o}$ will be updated simultaneously with other parameters during training.

DAG-LSTM Unit. During the process of iteratively generating and classifying, for each candidate, we want to embed all informations of that candidate into a hidden state. Then the hidden state can be used for classification, or as the representation of that candidate to be operand of other candidates. So we use the hidden states of operands and the embedding of operator to compute a candidate's hidden state. And LSTM is well suited to this work.

The input for a relation candidate $r_t = (l, o, r)$ includes the hidden state h_l of the left operand , h_r of the right operand, and an embedding o_t of the operator. The following equations compute the hidden state of r_t:

$$
\begin{aligned}
i_t &= \sigma\left(W_i o_t + U_i\left[h_l, h_r\right] + b_i\right) \\
f_t^r &= \sigma\left(W_f^r o_t + U_f^r\left[h_l, h_r\right] + b_f^r\right) \\
f_t^l &= \sigma\left(W_f^l o_t + U_f^l\left[h_l, h_r\right] + b_f^l\right) \\
o_t &= \sigma\left(W_o o_t + U_o\left[h_l, h_r\right] + b_o\right) \\
\hat{c}_t &= \tanh\left(W_c o_t + U_c\left[h_l, h_r\right] + b_c\right) \\
c_t &= f_t^r \odot c_r + f_t^l \odot c_l + i_t \odot \hat{c}_t \\
h_t &= \tanh(c_t) \odot o_t
\end{aligned}
\tag{3}
$$

where σ denotes the logistic function, $\odot$ denotes element-wise multiplication, W, U and b are weight matrices and bias vectors, and subscripts i, f, o, c indicate different parameters. Conventionally we call i_t, f_t, and o_t as input, forget, and output gate.

This computation inherits the spirit of sequential LSTM which uses memory cells and gates to transfer long-term information. The difference is that we use two separate forget gates for the left child and right child respectively [25, 29] .

To do classification, we apply a linear transformation followed by softmax function on h_t: $s_t = \text{softmax}(W_s h_t)$, to predict whether the node is positive or not.

Our method fuse the information of operator into the output hidden vector, and use an universal binary classification for all operators. This approach should alleviate the subsequent relation extractions in the following layers. There is another approach that only use operands to compute the hidden state, and feed that hidden state to a binary classification specified by o. The idea behind our approach is that by fusing the information of the children with the information of the operator explicitly, we allow the operator information to be transferred throughout the DAG structure by hidden states.

Tracking hidden states. To put all the DAG-LSTM units in one sentence together, we keep a matrix $\boldsymbol{H}$ to stores all nodes' hidden state. First, $\boldsymbol{H}$ records all hidden state of tokens returned from Bi-LSTM. Then, iteratively, after predicting all candidates in one layer, the hidden states of positives are appended to $\boldsymbol{H}$ and become available for the subsequent relations. Saving only the hidden states of the previous layer is not enough because a relation can have operand from any lower layers.

Go through the process by an example in Fig. 5. At the beginning of DAG-LSTM, we have $\boldsymbol{H}$=[h_1(In), h_2(2012), h_4(,), h_5(tax payables), h_6(constitutes), h_7(20%), h_8(of), h_9(total liabilities), h_{10}(.)].

At layer 1, there are two candidates c_1:(tax payables, @, 2012), c_2:(total liabilities, @, 2012). For the first one, using h_5, h_2, $\boldsymbol{E_o}(@)$ as input, compute the hidden state and classify. The same process for the second candidate. Assume we predict them as positives. Then we append h_{11}(for c_1), h_{12}(for c_2) to $\boldsymbol{H}$.

At layer 2, many candidates are generated but their operands are r_1 (c_1 in layer1), r_2 (c_2 in layer1) and 20%. We get their hidden vectors h_{11}, h_{12}, h_7 from $\boldsymbol{H}$ respectively and compute the hidden vectors of candidates and classify. Assume only $c_3 : (r_1, \div, r_2)$ is positive. We append its hidden vector h_{13} to $\boldsymbol{H}$.

At layer 3, all the hidden vectors required are $h_{11}, h_7, h_{12}, h_{13}$. Only $c_4 : (20\%, =, r_3)$ is positive. We append its hidden vector h_{14} to $\boldsymbol{H}$. Since no arithmetic relations are positive, the process finishes.

4 TRAINING AND SPEEDUP

4.1 Data Preparation and Loss Function

During training, we have the ground truth data: the DAG structure of formulas. These are positive nodes in DAG. Other possible relation candidates are negative nodes. If we train a model to predict all candidate nodes correctly, we could ensure that the model will extract all formulas from sentence correctly. Therefore, these positive and negative nodes are enough to train a reliable model for prediction.

The loss function is defined as the overall cross entropy. Suppose all sentences we have are S. Each sentence s^i have a candidate set C^i that includes both positive and negative relation candidates in the DAG. C^i contains n^i candidates: $c_1^i, ..., c_{n^i}^i$. Their labels are $y_1^i, y_2^i, ..., y_{n^i}^i$, and the probabilities of being positive from model are $p(c_1^i), p(c_2^i), ..., p(c_{n^i}^i)$. Then the loss function is:

$$L = -\sum_{s^i \in S} \sum_{c_j^i \in C^i} y_j^i \log(p(c_j^i)) + (1 - y_j^i) \log(1 - p(c_j^i))$$

4.2 Speedup

The slow training speed is a bottleneck for the development of neural networks with complex input structure like our model. Especially, current implementations compute one example at a time which is poorly suited for GPU computation [5]. We propose methods that parallelize computation inside one sentence by layer, and parallelize among sentences. They are shown in Fig. 7.

One sentence might have thousands of relation candidates. The computed hidden state of one relation or entity might be the input

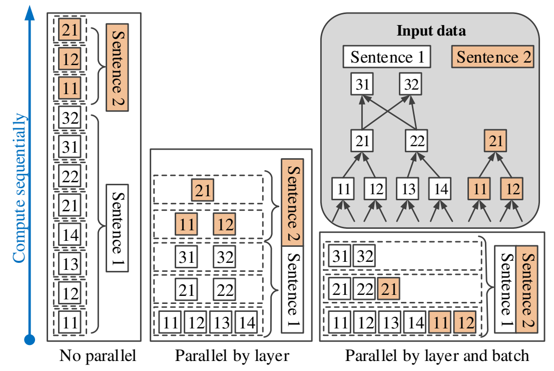


Figure 7: Parallelization methods.

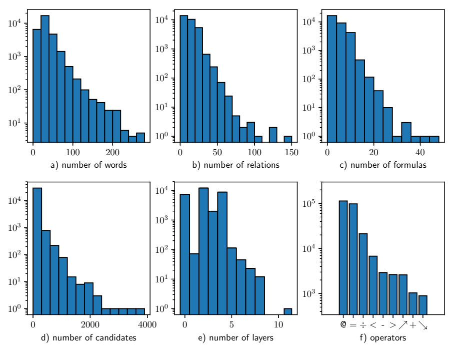


Figure 8: Distribution of sentences and relations.

of another relation. So during training, we regard all candidates in one sentence as one example and compute them in one pass.

Parallel by layer in one sentence. As Fig. 5 shows, all candidates in the same layer share the same hidden states from lower layers. Therefore, one layer of candidates can be computed in parallel.

Parallel by layer and batch Most current implementations of tree-LSTM in Theano or Torch have to compute one example at a time [4]. Besides computing all candidates in the same layer in one sentence at a time, we compute all candidates in the same layer for all sentences in a batch in parallel.

5 EXPERIMENT

5.1 Data and settings

We collected a dataset consisted of tens of thousands of labelled sentences. Since understanding financial text requires domain knowledge, we trained several people to construct this dataset. Each sentence is labelled by three people to minimize the labelling errors. Fig. 8 gives some statistic about our dataset. Distributions of sentences with regard to number of words, relations, formulas, candidates and layers are shown in a) to e). Most sentences have 20-40 words, less than 50 relations, less than 10 formulas, up to hundreds of candidates, and 2 to 5 layers. But some sentences have more than 200 words, hundreds of relations, and thousands of candidates. Subfigure f) depicts the number of relations of each operator. "@", "=", and "÷" are the most common operators where "+", ">" are about ten times rarer than them. The operators are very imbalanced. And the difference between the number of candidates and relations also indicates the imbalance between positive and negative samples.

The model is implemented in Theano [26]. We only keep 3500 most frequent tokens and the rest are represent by a special UNKNOWN token. We also assign each type of entities (time, value, and indicator) a special token. The word embedding size is 128. Bi-LSTM doubles the size of hidden state of each token. And the hidden size of relation is also 256. We use Adadelta [27] as our optimizer. The batch size is 16.

[4]https://github.com/stanfordnlp/treelstm
https://github.com/dasguptar/treelstm.pytorch
https://github.com/ofirnachum/tree_rnn

Table 1: Results (%) on test set, with or without operator embedding.

	Relation			Formula			Sentence
	P	R	F_1	P	R	F_1	accuracy
With	97.78	98.33	98.06	96.53	96.45	96.49	92.02
Without	97.28	97.17	97.23	97.17	94.58	95.84	87.88

From the collected data we find descriptions about "growth rate" are popular. If we only use $\{+, -, \times, \div\}$, they require two relations. To reduce the layer number, we add special operators "$\nearrow$", $a \nearrow b = (b - a)/a$. Similarly, we add "$\searrow$" for "decline rate".

5.2 Effective evaluation

We evaluate the result in several ways to the performance at different levels. At relation level, we calculate the precision (P), recall (R), and F1-score (F_1). At formula level, we also report these metrics where precision is defined as (number of formulas we predict correctly) / (number of formulas in ground truth). At sentence level, we calculate accuracy, the percentage of sentences whose formulas are all correctly extracted. The relation and sentence level metrics are more directly related to user experience for NCC.

To evaluate the effectiveness of our model, we compare it with a model that does not fuse the operator embedding into the hidden vector. That means when computing hidden vectors of one relation in Equation 3, we omit the $(W \cdot o_t)$ term. The evaluation result in Table 1 shows that using operator embedding will improve the model performance, especially at formula and sentence level. Model with operator embedding predicts correctly on 92.02% of sentences, which is a very promising result.

Moreover, we analyze the model performance with regard to the number of words, formulas, relations, etc. which showed similar patterns. Fig. 9 illustrates the performance with regard to the number of relations in a sentence. Sentences are grouped by relation number. Blue solid line is the sentence level accuracy on different groups. Red dashed line is the percentage of sentences in that group in test data. There is a trend that the model is prone to make more errors on sentences with more relations (long sentences). There

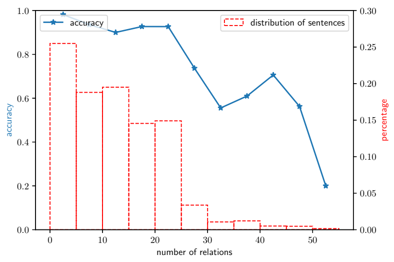


Figure 9: Accuracy on each group.

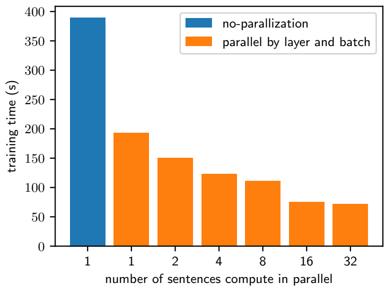


Figure 10: The average training time in one epoch.

are two possible reasons: long sentences are hard intrinsically, and long sentences are relatively rare in training data. Only 1.24% of sentences have more than 40 relations, but the average accuracy in these groups is around 60%, which is still a non-trivial result (accuracy drops sharply for sentences with more than 50 relations, but there are only 11 sentences in that group).

5.3 Case Study

Fig. 11 gives some cases that the model prediction contains mistakes, which illustrates the typical problems we are facing. The detailed analysis on each case is shown in figure and we summarize as follows: a) the verbal expression might be ambiguous and confuse the model, but this should be avoided in formal documents; b) performance on some operators is worse than others, that should attribute to the lack of training data (in training data, "+, >" are about ten times rarer than "=, ÷"); c) model might make mistakes when the expression is complicated; d) there are some complicate linguistic phenomenons hard to handle.

5.4 Speedup Result

Fig. 10 shows the training time of one epoch on ten thousand sentences. If no parallelization is applied, the average running time is 389.38s. Parallelizing by layer (parallel by layer and batch, where batch size=1) reduce the time to 193.00s. If we parallelize by layer and batch, and set batch size to 2, 4, ..., the training time keeps falling, and reaches 72.50s when batch size is 32. Comparing to 389.38s, it saves 81.36% of the training time, which means more than 5 times speedup. The result shows that methods we proposed reduce the training time significantly.

6 RELATED WORK

We introduce the related work in three aspects: some related NLP tasks, neural network models, and speeding up methods.

Related NLP tasks. The the most related tasks include relation extraction and semantic parsing. Relation extraction has a long history. Most works focus on relations between two entities [1, 18, 19]. A recent work "numerical relation extraction" [17] try to extract relations that contain numbers such like "inflation rate(India, 10.9%)". They stop at only one layer of relation, while our problem asks to take a step forward to extracting "relation over relation", which is more difficult considering the complexity of structure.

Semantic Parsing is a wide concept [4] which maps a natural-language sentence into a formal representation of its meaning. Our work is a type of semantic parsing that map natural language to the formula structure. Other semantic parsings include mapping natural language to database queries for question answering [3, 4, 15]. And Hershcovich et al. [11] mapped natural language to a predefined UCCA structure using transition-based parsing. By adding more operations to transition set, it solved the reentrancy, discontinuous, non-terminal problems. But in that work, every token in a sentence is a part of the final structure which is different with our problem and transition-based method is not suited for our problem due to the long distance relations.

A closely related problem is so-called "equation parsing" which extract the equation structure in a sentence. There are some preliminary works in this field. They assume one sentence contains one equation (our model can handle multiple formulas), and the equation is a tree structure while our problem need to extract DAG structure. Specifically, Roy et al. [21] extracted all "triggers" followed by predicting the structure based on Cocke–Younger–Kasami algorithm combined with SVM. It assume the structure to be projective. Koncel-Kedziorski et al. [13] applied integer linear programming to generate candidate tree structures then chosen one of them with SVM.

Model structure. From the perspective of model structure, our work combines sequential LSTM and structured LSTM, and shares some basic idea with recursive neural network. Bidirectional LSTM is effective to represent tokens in its sentential context [9, 12]. It has been widely used in many works such as semantic parsing [11], relation extraction [19], neural machine translation [2, 6] etc.

Structured LSTM started from tree structure, and developed repidly to DAG strcture. Tree-LSTM extend LSTM from sequential to structural. There are many variations of Tree-LSTM which share the same idea but are slightly different in the calculation detail depending on the task. Leveraging the tree structure information of sentence could outperformed state of the art models in sentiment classification [25, 29]. And it is also applied in relation extraction [19] recently. DAG structured LSTM has been proposed recently and applied to many fields, like using DAG to represent the different semantic composition [28], using DAG RNN to connect all pixels in one image for scene labeling [23]. But most of tree and DAG structured LSTM concentrated on incorporating structure information as input to model to get a better representation, while in our work we use DAG-LSTM both to get a good representation and to do structure prediction.

	Chinese	2014年度、2015年度、2016年度，发行人发生的销售费用分别为56,073.37万元、57,940.22万元、63,217.70万元，占发行人同期营业收入的比重分别为3.61%、2.26%、2.70%，稳定下降。
a)	English	In year 2014, 2015 and 2016, the selling expenses were ￥560.73 million, ￥579.40 million and ￥632.18 million respectively, constituted 3.61%, 2.26% and 2.70% of business incomes of the corresponding periods, dropping steadily.
	Errors & Analysis	(selling, @, 2014) > (selling, @, 2015) and (selling, @, 2015) > (selling, @, 2016) appear in label result, but not in prediction. Notice the expression in this sentence is actually ambiguous: does the ratios drop or both expenses and ratios drop ?
	Chinese	公司2014年末较2013年末投资性房地产账面价值增加463,998.98万元，增长了72.24%，主要是购置了房屋建筑物、土地使用权共计446,830.94万元，以及原投资性房地产公允价值变动增加17,168.04万元。
b)	English	At the end of 2014, the book value of investment real estates increased ￥4,639.99 million than 2013, the growth rate was 72.24%, mainly because purchased buildings, land use rights ￥4,468.31 million in total, and the fair value of investment real estates increased ￥171,68 million.
	Analysis on Errors	(buildings, @, 2014) + (land use rights, @, 2014) = ￥4,639.99 million appear in label result but not in prediction. That might because "+" operator is relatively rare in training dataset.
	Chinese	2012年度、2013年度、2014年度和2015年1-3月，公司财务费用有所波动，2013-2015年(2013年、2014年、2015年)，公司财务费用分别为957.23万元、-1,411.25万元和1,468.40万元，2014年因市场利率整体较高，当年公司货币利息收入较多导致财务费用为负；同期财务费用占营业收入的比重分别为0.35%、-0.47%和0.34%。
c)	English	In 2012, 2013, 2014 and the first quarter of 2015, financial expenses fluctuated slightly, in year 2013, 2014 and 2015, financial expenses were ￥9.57 million, ￥-14.11 million and ￥14.68 million respectively, in 2014 the average interest rate were high, therefore higher company monetary interest revenue made financial expenses negative; financial expenses constituted 0.35%, -0.47% and 0.34% of business income of the corresponding periods.
	Analysis on Errors	In prediction it missed the ratios. That might because the sentence is long, and there are some not closely related expressions before the last words.
	Chinese	2012年度、2013年度、2014年度和2015年1-3月，公司管理费用分别为8,215.12万元、14,288.53万元、18,006.34万元和15,437.10万元，占当期营业收入比例分别为1.89%、1.13%、0.82%和1.05%，公司管理费用金额波动上升而占当期营业收入比例有所下降，说明公司管理水平和费用控制能力较好。
d)	English	During year 2012, 2013, 2014 and the first quarter of 2015, G&A expenses (p_1) are ￥82.15 million, ￥142.89 million, ￥180.06 million and ￥154.37 million respectively, constituted 1.89%, 1.13%, 0.82% and 1.05% of business incomes (p_2), G&A expenses (p_3) increased and the proportion to business incomes (p_4) of the corresponding period decreased, that indicated a high level of management and cost control ability.
	Analysis on Errors	p_1 and p_3 have the same meaning but is actually different node in DAG structure. In prediction model output several wrong relations like (p_3, @, 2012) / (p_2, @, 2012) < (p_3, 2013) / (p_4, 2013). Although the underlying meaning is correct, what we expected is (p_3, @, 2012) / (p_4, @, 2012) < (p_3, @2013) / (p_4, @, 2013). This is a common linguistic phenomenon "co-reference", which might require some further effort to address in our problem setting.

Figure 11: Some examples that the model prediction is not consistent with label.

Recursive neural network [24] is proposed for image segment and natural language parsing in a recursive way. The overall framework shares insight with ours, but there are some differences. First, our model want to understand the structure of formulas and may end up with multiple roots, while recursive NN want to get a global idea of one sentence or image and merged into one root finally. Second, recursive NN "merge" two nodes into one parent and think the parent contains the information of its children, so the children are removed from the subsequent process which is clearly not compatible with the reuse characteristic of formulas.

Speeding up neural network models. The wild usage of complex neural network in natural language processing has proposed an urgent demand for faster training speed. Moshe et al. [16] treats neural network as a computation graph, and computes all nodes with same "depth" in graph in parallel. That work implemented in tensorflow shows that the parallelization method has a big speedup power on tree input. Our work are built on Theano and shows significant speedup on DAG structure. Our method is build on pure Theano without additional wrappers, which shows it is not too complex to extend an implementation from sequential computing to parallel computing.

7 CONCLUSION

When investors find out inconsistent errors in disclosure documents, they may doubt the authenticity of the whole document, which can be quite a great cost to the firms at some time. In order to detect these accounting errors more effectively, this paper proposes the iterative relation extraction model to extract formula from text. This is a key component of our automatic Numerical Cross-Checking (NCC) platform. By defining formula as a DAG structure, our model is able to extract the DAG with high accuracy. Based on this model, out system is able to detect numerical inconsistencies in real applications and has become widely accepted in the Chinese financial community. The development of this system will be ongoing.

ACKNOWLEDGMENTS

The research work was supported by the National Natural Science Foundation of China under Grant No. 61473274, the National Key Research and Development Program of China under Grant No. 2017YFB1002104. Jiaquan Yao was partially supported by the National Natural Science Foundation of China under Grant No. 71502152.

REFERENCES

[1] Charu C Aggarwal and ChengXiang Zhai. 2012. *Mining text data*. Springer Science & Business Media.

[2] Dzmitry Bahdanau, Kyunghyun Cho, and Yoshua Bengio. 2015. Neural machine translation by jointly learning to align and translate. *ICLR* (2015).

[3] Jonathan Berant, Andrew Chou, Roy Frostig, and Percy Liang. 2013. Semantic Parsing on Freebase from Question-Answer Pairs.. In *EMNLP*.

[4] Phil Blunsom, Nando de Freitas, Edward Grefenstette, and Karl Moritz Hermann. 2014. A deep architecture for semantic parsing. In *ACL Workshop on Semantic Parsing*.

[5] Samuel R. Bowman, Jon Gauthier, Abhinav Rastogi, Raghav Gupta, Christopher D. Manning, and Christopher Potts. 2016. A Fast Unified Model for Parsing and Sentence Understanding. In *ACL*.

[6] Huadong Chen, Shujian Huang, David Chiang, and Jiajun Chen. 2017. Improved Neural Machine Translation with a Syntax-Aware Encoder and Decoder. In *ACL*.

[7] Preeti Choudhary, Kenneth J Merkley, and Katherine Schipper. 2016. Qualitative characteristics of financial reporting errors deemed immaterial by managers. (2016).

[8] Vivian W Fang, Allen H Huang, and Wenyu Wang. 2017. Imperfect accounting and reporting bias. *Journal of Accounting Research* (2017).

[9] Alex Graves, Navdeep Jaitly, and Abdel-rahman Mohamed. 2013. Hybrid speech recognition with deep bidirectional LSTM. In *Automatic Speech Recognition and Understanding (ASRU)*.

[10] Naeemul Hassan, Fatma Arslan, Chengkai Li, and Mark Tremayne. 2017. Toward Automated Fact-Checking: Detecting Check-worthy Factual Claims by ClaimBuster. In *KDD*.

[11] Daniel Hershcovich, Omri Abend, and Ari Rappoport. 2017. A Transition-Based Directed Acyclic Graph Parser for UCCA. *ACL* (2017).

[12] Eliyahu Kiperwasser and Yoav Goldberg. 2016. Simple and accurate dependency parsing using bidirectional LSTM feature representations. *TACL* (2016).

[13] Rik Koncel-Kedziorski, Hannaneh Hajishirzi, Ashish Sabharwal, Oren Etzioni, and Siena Dumas Ang. 2015. Parsing algebraic word problems into equations. *ACL* (2015).

[14] Alastair Lawrence. 2013. Individual investors and financial disclosure. *Journal of Accounting and Economics* (2013).

[15] Chen Liang, Jonathan Berant, Quoc Le, Kenneth D Forbus, and Ni Lao. 2017. Neural symbolic machines: Learning semantic parsers on freebase with weak supervision. *ACL* (2017).

[16] Moshe Looks, Marcello Herreshoff, DeLesley Hutchins, and Peter Norvig. 2017. Deep Learning with Dynamic Computation Graphs. *ICLR* (2017).

[17] Aman Madaan, Ashish Mittal, G Ramakrishnan Mausam, Ganesh Ramakrishnan, and Sunita Sarawagi. 2016. Numerical Relation Extraction with Minimal Supervision.. In *AAAI*.

[18] Mike Mintz, Steven Bills, Rion Snow, and Jurafsky Dan. 2009. Distant supervision for relation extraction without labeled data. In *ACL*.

[19] Makoto Miwa and Mohit Bansal. 2016. End-to-end relation extraction using lstms on sequences and tree structures. *ACL* (2016).

[20] Michèle B. Nuijten, Chris H. J. Hartgerink, Marcel A. L. M. van Assen, Sacha Epskamp, and Jelte M. Wicherts. 2016. The prevalence of statistical reporting errors in psychology (1985–2013). *Behavior Research Methods* (2016).

[21] Subhro Roy, Shyam Upadhyay, and Dan Roth. 2016. Equation Parsing: Mapping Sentences to Grounded Equations. In *EMNLP*.

[22] Mike Schuster and Kuldip K. Paliwal. 1997. Bidirectional recurrent neural networks. *IEEE Transactions on Signal Processing* (1997).

[23] Bing Shuai, Zhen Zuo, Bing Wang, and Gang Wang. 2016. Dag-recurrent neural networks for scene labeling. In *CVPR*.

[24] Richard Socher, Cliff C Lin, Chris Manning, and Andrew Y Ng. 2011. Parsing natural scenes and natural language with recursive neural networks. In *ICML*.

[25] Kai Sheng Tai, Richard Socher, and Christopher D Manning. 2015. Improved semantic representations from tree-structured long short-term memory networks. *ACL* (2015).

[26] Theano Development Team. 2016. Theano: A Python framework for fast computation of mathematical expressions. *arXiv* (2016).

[27] Matthew D. Zeiler. 2012. ADADELTA: An Adaptive Learning Rate Method. *arXiv* (2012).

[28] Xiaodan Zhu, Parinaz Sobhani, and Hongyu Guo. 2016. DAG-Structured Long Short-Term Memory for Semantic Compositionality. In *NAACL HLT*.

[29] Xiaodan Zhu, Parinaz Sobihani, and Hongyu Guo. 2015. Long short-term memory over recursive structures. In *ICML*.